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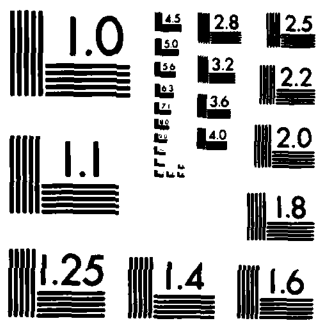

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Structures Technical Memorandum 385

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THE CONSTANT CURRENT STRAIN GAUGE BRIDGE

by

E.S. MOODY

SUMMARY

A simplified analysis is used to develop expressions for the output of the commonly used strain gauge bridge configurations with Constant Current excitation. Expressions for initial offset compensation, shunt calibration and the influence of lead resistance are developed. Consideration is given to some means for error correction.

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CONTENTS

1. INTRODUCTION

All the early electrical strain gauge bridge circuits employed constant voltage sources for bridge excitation. The technqiues developed for the classical direct-current and alternating-current component-measuring bridges were transferred to the strain gauge bridges with only minor modification.

With the introduction of the semi-conductor strain gauges the advantages to be gained by the use of constant current bridge excitation became apparent. However, the difficulties associated with the construction of suitable constant current circuitry inhibited its introduction on a large scale until the mid 1970's. The availability of inexpensive high-gain, integrated circuit amplifiers has simplified the task of producing practical constant current supplies, and by the end of 1979 extensive use of this form of excitation was being made in these Laboratories.

While the use of constant current sources does provide a bridge of enhanced stability, the network shares some of the problems of the constant voltage circuit and introduces a few of its own. In this paper an attempt has been made to present some of the formulae for the constant current bridge, to investigate the effects of lead resistance and to examine, for the simplest bridge, the influence of initial offset compensation (initial balance) on the sensitivity of the bridge to strain and on the bridge configurations to be used. → to back p.

2. BRIDGE FORMULAE

The expression for the output voltage (V_o) of the general Wheatstone network in Fig. 1, is established in Appendix 1. This development is for a simple, essentially equal-element bridge in which only small changes in resistance take place. However, the procedure used is quite general and appropriate for elements of any value. An infinite input-impedance detector is assumed.

The expression for the bridge output voltage is:

$$V_o = \left[\frac{[\ a(1+d) + d\ ] - [\ b(1+c) + c\ ]}{4 + a + b + c + d} \right] \times [I.R.] \qquad ..(1)$$

Where I is the constant current injected into the circuit at node 1 and R is the initial, undisturbed value of each bridge element, usually referred to as the bridge resistance, and a,b,c and d are small fractional departures of each element from the value R. From the above expression, by selecting appropriate values for a,b,c and d, the formulae for the output voltages of the commonly used strain gauge configurations can be obtained. These formulae, together with the corresponding expressions for the equivalent constant voltage circuits are set out in Table 1.

There are several points to be noted about the constant current formulae:-

[a] The output voltage, V_o, is directly proportional to bridge resistance.

[b] For the three possible two-active-element configurations the output voltage is the same.

[c] Comparison of constant current and constant voltage expressions shows the improvement in linearity achieved with constant current. However, the nonlinear term is present in both bridges and, as Troke(2) has pointed out, this nonlinearity must be taken into account in all cases where shunt calibration techniques are used to establish strain sensitivity.

[d] Where all elements of the bridge are equally active, or where two adjacent elements across the supply are equally active (the changes in both cases being additive) the formulae for constant voltage are direct equivalents of the formulae for constant current.

The appearance of the bridge resistance in the expression for the output voltage is of concern to users of constant current bridges. Fortunately, the change in this term with temperature and with time is small, and its influence on bridge output voltage is generally negligible. In a good quality strain gauge bridge R may change by 0.04 ohms in 120 for 100 degrees Celsius change in temperature. This will change the output by about 0.03% for 100 degrees temperature change. It is of some interest to note that the expression for the short-circuit output current of the constant current bridge is independent of bridge resistance.

At the present time, current-sensitive detectors are out of favour for strain gauge bridges and, in any case, would need to be located at the bridge to avoid lead resistance effects. Systems providing signal conversion at the gauge site are currently receiving attention, and perhaps current detectors may be restored to favour in the future.

3. THE INITIAL OFFSET

Strain gauge bridges, particularly those used in structural testing as distinct from those in load cells, are rarely perfectly balanced, and a significant offset voltage will be present in the output. Before mini and microcomputers were introduced to strain gauge instrumentation an initial balancing circuit was considered an essential feature of the complete bridge network.

There were two major drawbacks to the initial balance circuit. Firstly, even when large and costly components were used, random jumps in the initial output could not be completely eliminated. Secondly, this circuit places shunting resistances of indeterminate value across two adjacent arms of the bridge thereby upsetting the thermal symmetry and, for a four-active-element bridge, reducing the bridge's sensitivity to strain by an incalculable amount.

The computer can provide automatic correction for any initial offset, and with the very long scale length of modern voltmeter-detectors, minimization of initial offsets is only rarely required. When some adjustment is necessary the degree of precision required is low and may be achieved by shunting one element of the bridge with a stable, fixed resistor. The value of resistance to be used can be determined most readily using a decade resistance box.

It should be clear that a shunt across an active element will reduce the sensitivity of the effective total element to any strain induced resistance change, but it may not be apparent that any shunting of a ratio-arm to achieve initial offset compensation may also bring about significant change in sensitivity to strain. Examination of equations (8) to (11) in Appendix 1, and the sensitivity ratios set out in Table 2, will show that the influence of the initial offset compensation on the bridge output varies with the compensation used. It can also be seen that for the worst case the loss of sensitivity is 0.5 percent. For purposes of illustration, it has been assumed that the bridge of Fig. 1 has three equal elements A, C and D and that element B is either larger of smaller.

For the above conditions there are two options available for initial offset compensation:-

[1] Where B is larger than the other elements, element D may be increased or element C decreased.

[2] Where B is smaller than the other elements, element D may be reduced or element C increased.

In equations (8) and (11), where offset compensation involves adjustment to element C, the initial fractional offset value 'b' does not appear in the numerator, and appears only as 'b^2', a very small number, in the denominator. However, in equations (9) and (10) where offset compensation is achieved by making adjustments to element D the fractional offset value appears in the numerator, and in the denominator it is doubled. As expected, the results in Table 2 show that when equations (8) and (11) apply, the change in sensitivity are negligible, but can be as high as 0.5 percent when equations (9) and (10) apply. If sensitivity changes of this magnitude cannot be tolerated, adjustments for both signs of offset in element B must be made in element C, either a shunt across it or a small resistance in series with it.

Clearly where the strain sensitivity is determined only after the offset adjustments have been made the above problem does not arise. Unfortunately, calibration after initial offset compensation is not always possible, and if accuracy is paramount, a calculation of the change should

be made. In a practical bridge elements A and B may be equal, and C and D equal to one another but not equal to A and B. Setting nominal values of elements A and B equal to R_1, and C and D equal to R, and then substituting in equation 1 of Appendix 1, expressions for bridge output voltage for compensated and uncompensated bridges can be determined. Using the symbol δ for those small fractional changes from the mean value resulting from strain on the element and setting $a = +\delta$, $b = +b$, $c = -b/1+b$, $d = \phi$:-

$$V_\delta = \frac{IR_1R\delta}{2(R + R_1) + \delta R_1 + \dfrac{b(R_1 - R) + b^2R_1}{1 + b}} \quad ..(2)$$

For $a = +\delta$, $b = -b$, $c = \phi$, $d = -b$:-

$$V_\delta = \frac{IR_1R\delta(1 - b)}{2(R + R_1) + \delta R_1 - b(R + R_1)} \quad ..(3)$$

When the bridge has no offset compensation the output for $a = \delta$, $b = +b$, $c = \phi$, $d = \phi$ is:-

$$V_{\delta 1} = IR_1R\left[\frac{\delta - b}{R_1(2+\delta+b)+2R} + \frac{b}{R_1(2+b)+2R}\right] \quad ..(4)$$

For $a = \delta$, $b = -b$, $c = \phi$, $d = \phi$:-

$$V_{\delta 1} = IR_1R\left[\frac{\delta + b}{R_1(2+\delta-b)+2R} \qquad \frac{b}{R_1(2-b)+2R}\right] \quad ..(5)$$

With these four equations and a knowledge of the resistance in the bridge prior to compensation an accurate assessment of the effect of compensation on strain sensitivity can be reached.

4. OFFSET COMPENSATION AND SHUNT CALIBRATION

If the signal from shunt calibration is to be the basis for strain calculations and not simply a check on system sensitivity the element, or for bidirectional calibration, elements shunted should not include the element used for initial offset compensation. For a bridge with a single active branch containing one or two active elements both offset compensation and uni-directional shunt calibration may be performed across the ratio arms thereby avoiding the problems introduced by lead resistance. In the general case, however, initial offset compensation should be applied in one branch and shunt calibration in the other.

5. SHUNT CALIBRATION AND LEAD RESISTANCE

The errors arising from the presence of significant lead resistance, or from lead resistance change, are well known, and demonstrated in Graph 1.

A general expression for the output voltage produced by shunting one element of a constant current bridge with a resistance R_{CAL}, in the presence of lead resistance, R_ℓ, as shown in Fig. 2. is developed in Appendix 2. This output voltage is:-

$$V_{OC1} = \frac{I\,[R^2 + 4R_\ell\,(R + R_\ell)]}{4[\frac{3R}{4} + 2R_\ell + R_{CAL}]} \qquad ..(6)$$

When R_ℓ is set to zero in this expression it reduces to:-

$$V_{OC2} = \frac{IR^2}{4\,[\frac{3R}{4} + R_{CAL}\,]} \qquad ..(7)$$

Equations (6) and (7) can be combined to provide an expression for the percentage change in the shunt calibration signal with change in lead resistance.

$$\left[\frac{V_{OC1}}{V_{OC2}} - 1\right] 100 = \left[\frac{[R^2 + 4R_\ell(R + R_\ell)][\frac{3R}{4} + R_{CAL}]}{R^2 \; [\; \frac{3R}{4} + 2R_\ell + R_{CAL}]} - 1\right] 100 \; . \qquad ..(8)$$

This can be reduced to an expression independent of the value of R_{CAL} by discarding $2R_\ell$ from the denominator to give:-

$$\left[\frac{V_{OC1}}{V_{OC2}} - 1\right] 100 = \frac{4R_\ell(R + R_\ell)}{R^2} \; . \qquad ..(9)$$

While the assumption of symmetrical lead resistance, and lead resistance change is not always valid the approximation is sufficiently good for all practical installations.

In Graph 1, the percentage change in calibrate signal, C, is plotted as a function of R over a range of values from 0.2 to 2.0 ohms. From this graph it is clear that lead resistance values should not exceed 0.1 ohms if this effect on calibrate signal is to be ignored. It would be very difficult to hold lead resistance to this low value, and a reasonable conclusion from Graph 1 is that shunt calibration is best avoided in 4-wire bridges; if shunt calibration is unavoidable, a 6-wire bridge should be used. Only the 6-wire circuit, or a 7-wire arrangement for bi-directional calibration, will completely eliminate these lead effects. Fortunately, with the constant current supply, wire diameters can be reduced to compensate for the additional wires. As a last resort, the resistance of the lead can be estimated and a correction applied.

6. OFFSET COMPENSATION AND LEAD RESISTANCE

The formulae developed for the shunt calibrate circuit can also be applied to the initial offset compensator where it is being shunted across a remote element. In this case changes in the lead resistance will induce small changes in the offset correction and appear as drift in the residual initial output voltage. In this case the calculated percentage change in the calibrate signal becomes the

percentage change in the compensating voltage. If unknown, the size of the initial offset, and hence the size of the compensating signal can be approximated from a knowledge of the compensating resistor. The percentage change for lead resistance change is applied to the compensating signal, and clearly the larger the initial offset the larger will be the drift from lead resistance.

For the single active branch bridge, with compensation across or in series with one of the ratio arms, lead effects on compensation can be ignored. Where the 4 elements of the bridge are installed at the measuring site series offset compensation would require the installation of small resistors adjacent to the gauge and as close to it as possible. The series resistor cannot be mounted remotely because this would introduce significant amounts of temperature sensitive copper lead wire into one bridge element and destroy the thermal stability of the bridge. If the gauge is not accessible, and in most structural testing applications it will not be, one arm of the bridge will require to be shunted when offset compensation is essential.

To minimize lead effects for remote shunt offset compensation this shunt must be provided with separate leads as for the calibrating shunt. This requirement could increase the total number of leads to 8 or, for bi-directional calibration, 9 wires.

7. THE 7-WIRE BRIDGE

Bi-directional shunt calibration is very rarely required for gauge systems used in structural testing. Therefore, the maximum number of wires required for a bridge in this application will be eight, four for bridge supply and signal output, two for the shunt calibrate resistor and two for an offset compensator if it is needed.

In those situations where the shunt calibrate resistor and the shunt compensator are applied across adjacent elements in an 8-wire configuration, or where the requirements can be adapted to place these shunts across adjacent elements, one lead wire may be shared and the bridge reduced to a seven-wire circuit.

In Appendix 3 the equivalence of the 8-wire and 7-wire bridges is studied, and it is shown that errors, if any, resulting from the discarding of one lead wire is negligible. The shunt calibrate signal is the same for both 7-wire and 8-wire bridges. However, the influence of offset compensation on bridge sensitivity should be kept in mind when trying to bring the shunting resistors to adjacent elements.

8. CONCLUSION

Constant current excitation provides a stable strain gauge system but a six-wire or eight-wire bridge must be used if shunt calibration procedures are employed and a high level of accuracy is required. The large number of wires per bridge can aggravate installation and trouble-shooting problems.

An alternative to the multi-wire bridge is the placing of bridge conditioning and detecting circuits so close to the gauge measuring sites that lead resistance can be neglected. From Graph 1, it can be seen that for a 4-wire constant current bridge with shunt calibration the lead resistance must be less than 0.1 ohms if corrections to the strain-equivalent-of-calibrate signal are not to be made. Four metres of 20 SWG solid copper wire, or 2.4 metres of 14/0.193 mm stranded hook-up wire will produce a resistance of 0.1 ohms. This is a relatively short length of wire in most gauge installations, and if the wires must be longer, either 6 wires should be used or a computed correction applied.

REFERENCES

[1] TROKE, Robert W. "Improving Strain Measurement Accuracy When Using Shunt Calibrations". Experimental Mechanics, pp. 397-400, October 1976.

APPENDIX 1

A common procedure for bridge analysis will be used throughout these Appendices and it is set out in some detail here. The general equations for the network of Fig. 1 are:-

$$I_1 + I_2 = I$$

$$I_1 [R_A + R_B] = I_2 [R_C + R_D]$$

$$V_O = I_1 R_A - I_2 R_C$$

whence,

$$V_o = I \frac{\left[R_A(R_C + R_D) - R_C(R_A + R_B)\right]}{R_A + R_B + R_C + R_D}$$

$$= I \left[\frac{R_A R_D - R_C R_B}{R_A + R_B + R_C + R_D}\right] \quad . \qquad ..(1)$$

If we assume a nominally equal-element bridge in which each element differs from the nominal value by only a small amount, and this difference is expressed as a fraction of the nominal value, equation (1) can be written:-

$$V_o = I \frac{[R(1+a) \;\; R(1+d) - R(1+c) \;\; R(1+b)]}{R(4 + a + b + c + d)}$$

which in turn reduces to:-

$$V_o = \frac{IR\ [\ [a(1+d) + d] - [b(1+c) + c]\ ]}{4 + a + b + c + d} \quad . \qquad ..(2)$$

The fundamental strain gauge equation is:-

$$\delta = GF.\varepsilon .$$

Where GF = Gauge Factor, ε = Strain and δ = resulting fractional change in resistance.

By substituting appropriate values for a,b,c and d, Equation (2) yields the expressions for constant current bridge output contained in Table 1 as follows:-

1. For 2 adjacent elements with equal and opposite strain values: $a = \delta$, $b = -\delta$, $c = 0$, $d = 0$.
   Equation 2 becomes:

$$V_o = \frac{IR\,[\delta - (-\delta)]}{4 + \delta - \delta}$$

$$= \frac{IR\delta}{2} . \qquad ..(3)$$

2. For corresponding elements in the two branches subjected to equal strains of opposite sign: $a = \delta$, $c = -\delta$, $b = 0$, $d = 0$.
   Equation 2 becomes:

$$V_o = \frac{IR\,[\delta - (-\delta)]}{4 + \delta - \delta}$$

$$= \frac{IR\delta}{2} . \qquad ..(4)$$

3. For equal strains in alternate elements: $a = \delta$, $d = \delta$, $b = 0$, $c = 0$.
   Equation 2 becomes:

$$V_o = \frac{IR\,[\delta\,(1 + \delta) + \delta]}{4 + 2\delta}$$

$$= \frac{IR\delta}{2} . \qquad ..(5)$$

4. For a single active element: $a = \delta$, $b = 0$, $c = 0$, $d = 0$.
Equation 2 becomes:

$$V_o = \frac{IR\,[\delta\,(1+\phi) - \phi]}{4+\delta}$$

$$= \frac{IR\delta}{4+\delta}\ . \qquad ..(6)$$

5. For equal additive strains in the four elements:
$a = \delta$, $b = -\delta$, $c = -\delta$, $d = \delta$.
Equation 2 becomes:

$$V_o = \frac{IR\,[\,[\delta(1+\delta)+\delta] - [-\delta(1-\delta)-\delta]\,]}{4+\delta-\delta-\delta+\delta}$$

$$= IR\delta\ . \qquad ..(7)$$

Equation 2 may be used to investigate changes in element strain sensitivity with changes in initial offset compensation, but first it is necessary to establish values for c or d which will compensate for particular initial values of b.

1. Let b have the initial value +b, and let a=0, d=0. The bridge can now be balanced by shunting element C until $c = -b/1+b$. With this value substituted in Equation 2 it becomes:

$$V_o = \frac{IR\left[-\left[b\left(1-\frac{b}{1+b}\right)-\frac{b}{1+b}\right]\right]}{4+b-\frac{b}{1+b}}$$

$$= \phi$$

and the bridge is balanced.

2. With the same conditions as in 1 above the offset compensation could have been achieved by placing a small series resistance in element D to make d=+b. Equation 2 then becomes:

$$V_o = \frac{IR\,[+b - b]}{4 + b + b}$$

$$= \phi$$

and the bridge is balanced.

3. Let b have the initial value -b, and let a=0, c=0. The bridge can now be balanced by shunting element D until d=-b, or by increasing element C with series resistance until c = b/1-b. That compensation is achieved with d=-b should be obvious. When c = b/1-b, equation 2 becomes:

$$V_o = \frac{IR\left[-\left[-b\left(1 + \frac{b}{1-b}\right) + \frac{b}{1-b}\right]\right]}{4 - b + \frac{b}{1-b}}$$

$$= \phi\ .$$

When element A is strained without initial offset compensation the output from the bridge will be the algebraic sum of offset output plus strain output. Assuming c=0, d=0, a=±δ, b=+b, equation 2 can be used to derive an expression for the output produced by strain alone.

$$V_{\delta 1} = IR\left[\left[\frac{\pm\delta - b}{4 \pm \delta + b}\right] + \frac{b}{4 + b}\right]$$

and similarily when b = -b,

$$V_{\delta 1} = IR\left[\left[\frac{\pm\delta + b}{4 \pm \delta - b}\right] - \frac{b}{4 - b}\right]$$

For the two conditions of initial offset considered, namely b = ±b, four expressions for the output from an initially balanced bridge can be derived, one for each of the methods used to give initial offset compensation. They are as follows:

1. For $a = \pm\delta$, $b = +b$, $c = -b/1+b$, $d = \phi$. Equation 2 gives:

$$V_\delta = IR\left[\frac{\pm\delta - [\,b(1 - \frac{b}{1+b}) - \frac{b}{1+b}\,]}{4 \pm \delta + b - \frac{b}{1+b}}\right]$$

$$= \frac{\pm\, IR\delta}{4 \pm \delta + \frac{b^2}{1+b}} \quad . \qquad ..(8)$$

2. If initial offset compensation had been achieved for 1 above by adding resistance to element D insteady of shunting element C the values would be $a = \pm\delta$, $b = +b$, $c = \phi$, $d = +b$, and equation 2 gives:

$$V_\delta = IR\left[\frac{\pm\delta\ (1 + b) + b - b}{4 \pm \delta + b + b}\right]$$

$$= \frac{\pm\, IR\delta\ (1 + b)}{4 \pm \delta + 2b} \quad . \qquad ..(9)$$

3. For $a = \pm\delta$, $b = -b$, $c = 0$, $d = -b$, initial offset compensation is achieved by shunting element D and equation 2 gives:

$$V_\delta = IR\left[\frac{[\ \pm\delta\ (1 - b) - b\ ] - [\ -b\ ]}{4 \pm \delta - b - b}\right]$$

$$= \frac{\pm\, IR\delta\ (1 - b)}{4 \pm \delta - 2b} \quad . \qquad ..(10)$$

4. For $a = \pm\delta$, $b = -b$, $c = b/1-b$, $d = \phi$, initial offset compensation is achieved by adding resistance to element C instead of shunting element D. Equation 2 now gives:

$$V_\delta = IR\left[\frac{\pm\delta - [-b\ (1 + \frac{b}{1-b}) + \frac{b}{1-b}\ ]}{4 \pm \delta - b + \frac{b}{1-b}}\right]$$

$$= \frac{\pm IR\delta}{4 \pm \delta + \frac{b^2}{1-b}} \quad . \qquad ..(11)$$

To illustrate the change in sensitivity with initial offset compensation the ratio $V_\delta / V_{\delta 1}$ can be calculated. This has been done for equations (8) and (10) using the expressions:

$$\frac{V_\delta}{V_{\delta 1}} = \frac{(4 \pm \delta + b)\ (4 + b)}{2(2 + b)\ (4 \pm \delta + \frac{b^2}{1+b})} = C_1 \qquad ..(12)$$

for a = $\pm\delta$, b = +b, c = -b/1+b, d = ϕ; and

$$\frac{V_\delta}{V_{\delta 1}} = \frac{(4 \pm \delta - b)\ (4 - b)\ (1 - b)}{2(2 - b)\ (4 \pm \delta - 2\ b)} = C_2 \qquad ..(13)$$

for a = $\pm\delta$, b = -b, c = ϕ, d = -b. The results for several values of fractional resistance change in element R_A, a = δ and several values of the initial offset b for which compensation has been applied are tabulated in Table 2. Only the single-active-gauge bridge is examined because this is the most likely circuit to require initial offset adjustment, and because the equations describing this circuit are reasonably manageable.

APPENDIX 2

An expression for the output voltage from a simple bridge when one element is shunted by a fixed resistor is developed. The analysis is appropriate for the shunt calibration resistor, R_{CAL}, or for the shunt initial offset compensator, R_{SHNT}. Lead resistances are included and the justifiable assumption is made that all lead resistances are equal. The circuit is given in Fig. 2.

In Fig. 3a lead resistances which do not influence the output voltage have been removed and the layout changed to highlight the transformation to be made in order to arrive at the circuit of Fig. 3b.

For the general case where the elements are R_A, R_B, R_C and R_D, the expressions for circuit currents and voltages are cumbersome. However, by using the fractional forms R(1+a), R(1+b) etc. together with single symbols to represent complex expressions, the analysis is manageable.

From Fig. 3b by inspection:

$$[I - I_{CAL}]\left[R_\ell + \frac{R(1+a)(2+c+d)}{4+a+b+c+d}\right] = I_{CAL}\left[R_\ell + R_{CAL} + \frac{R(1+a)(1+b)}{4+a+b+c+d}\right].$$

By setting $4 + a + b + c + d = \Sigma$ this expression reduces to:

$$I_{CAL} = \frac{I\,[\,R_\ell.\Sigma + R(1+a)(2+c+d)\,]}{[\,(2R_\ell + R_{CAL})\Sigma + R(1+a)(1+b) + R(1+a)(2+c+d)\,]}\ .$$

Set the coefficient to I equal to γ, and the expression becomes:

$$I_{CAL} = I\,\gamma\ . \qquad ..(1)$$

From Fig. 3a by inspection:

$$I_2\,[\,R(1+c) + R(1+d)\,] = [\,I - I_{CAL} - I_2\,]\,R(1+a) + [\,I - I_2\,]\,R(1+b).$$

Substituting for I_{CAL} and using Σ this expression can be rearranged to give I_2:

$$I_2 = \frac{I}{\Sigma} [2 + a + b - \gamma(1+a)] . \qquad ..(2)$$

An expression for bridge output voltage can also be obtained by inspection of Fig. 3a:

$$V_o = I_2 R(1+d) - [(I - I_2) R(1+b) + I_{CAL} R_\ell] .$$

By subsitituting the values for I_{CAL} and I_2 given in equations 1 and 2, and letting $R_A = R_B = R_C = R_C = R$ such that $a = b = c = d = \phi$ this expression reduces to:

$$V_{OC} = \frac{2IR (2 - \gamma)}{\Sigma} - I[R + \gamma R_\ell] . \qquad ..(3)$$

To illustrate the effects of lead resistance on the shunt calibrate signal, or on the offset compensating voltage, equation 3 can be written:

$$V_{OC1} = - \frac{I[R^2 + 4R_\ell(R + R_\ell)]}{4[3R/4 + 2R_\ell + R_{CAL}]} . \qquad ..(4)$$

The reference value occurs when $R_\ell = 0$. The equation then becomes:

$$V_{OC2} = - \frac{IR^2}{4[3R/4 + R_{CAL}]} . \qquad ..(5)$$

Equations 4 and 5 can be combined to given an expression for the percentage change in shunt signal as the lead resistance increases from zero:

$$C = \left[\frac{[R^2 + 4R_\ell\ (R+R_\ell)]\ [3R/4 + R_{CAL}\ ]}{R^2\ [3R/4 + 2R_\ell + R_{CAL}\ ]} - 1 \right] 100 \qquad ..(6)$$

The percentages are shown graphically for lead resistance values up to 2 ohms in Graph 1.

Note that in equation 6, $2R_\ell$ is very much smaller than R_{CAL} and it should be a reasonable approximation to discard the lead resistance from the denominator. If this is done the expression becomes independent of R_{CAL}. The degree of dependence of the percentage change in the shunt calibrate signal on R_{CAL} is demonstrated using equation 6 for two values of R_{CAL}. The values obtained are set out in Table 3. When $2R_\ell$ is discarded from equation 6 the expression for percentage change can be reduced to:

$$C = \left[\frac{4R_\ell\ (R + R_\ell)}{R^2} \right] 100\ . \qquad ..(7)$$

APPENDIX 3

The circuits to be compared are in Fig. 4a, the complete 8-wire bridge, and in Fig. 4b, the shared-wire, 7-wire bridge.

When the shunt calibration circuit is open, the circuits of Figs. 4a and 4b are identical. As well as being the condition for normal operation, this is also the condition for initial offset compensation, and for equal values of offset 'a' and lead resistance $R\ell$ the value for R_{SHNT} will be the same in both circuits.

For initial offset compensation it was shown earlier that the value of the shunt, $R_{SHNT} + 2R_\ell$ across R must reduce the value of this bridge element to R(1-a/1+a) which can be simplified to R/1+a. Therefore, for both these circuits, initial offset compensation will be achieved when:

$$\frac{(R_{SHNT} + 2R_\ell)\,R}{(R_{SHNT} + 2R_\ell) + R} = \frac{R}{1+a} .$$

If $R_{SHNT} + 2R_\ell$ is replaced by r_2, a simple expression relating 'a', the offset, and r_2 the offset compensation is:

$$a = \frac{R}{r_2} . \qquad ..(1)$$

Fig. 4b shows the shared-wire, 7-wire bridge with the shunt calibrate circuit closed. This circuit can be transformed to that of Fig. 4c, which is of the same form as Fig. 4a, by performing a star-to-delta transformation on the shunting circuit. After transformation, the shunt calibrate circuit will have the values:

$$R_{CAL} + 2R_\ell + \frac{(R_{CAL} + R_\ell)\,R_\ell}{(R_{SHNT} + R_\ell)}$$

i.e. $$r_2 + \frac{(R_{CAL} + R_\ell)\,R_\ell}{(R_{SHNT} + R_\ell)} = r_2`$$

The shunt compensation circuit will have the values:

$$R_{SHNT} + 2R_{\ell} + \frac{(R_{SHNT} + R_{\ell})\ R_{\ell}}{(R_{CAL} + R_{\ell})}$$

i.e. $$r_1 + \frac{(R_{SHNT} + R_{\ell})\ R_{\ell}}{(R_{CAL} + R_{\ell})} = r_1'$$

and, as shown in Fig. 4c, a third component appears shunting the output and with a value given by:

$$\frac{(R_{CAL} + R_{\ell})\ (R_{SHNT} + R_{\ell})}{R_{\ell}} + R_{SHNT} + R_{CAL} + 2R_{\ell}\ .$$

The resistance of this output shunt is very high because it contains the product of two large resistances R_{CAL} and R_{SHNT} divided by the lead resistance which should never be greater than 1 ohm. Since it is only in shunt with the output, it can be discarded for this exercise.

By discarding the output shunt, Fig. 4c becomes identical in form to Fig. 4a, and from this it can be assumed that the circuit of Fig. 4b can be replaced by the circuit of Fig. 4a if r_1 and r_2 are replaced by r_1' and r_2'.

To determine the maximum differences between r_1 and r_1' and between r_2 and r_2', the maximum value for R_{CAL} and R_{SHNT} can be taken as 200 000 ohms; and the minimum values can be taken as 10 000 ohms. With R_{ℓ} at 1 ohm the above figures yield differences of 20.0 and 0.05. Notice that when the difference is a maximum, the value of r is also a maximum and when the difference is a minimum r is also a minimum. For the maximum difference, the correction is one part in 10^4 and for the minimum difference, it is one part in 20×10^4. While these small terms are unlikely to influence the output to a significant degree, numerical values have been determined for the four possible combinations arising from the values selected. The values of 'a' corresponding to

the values of R_{SHNT} are calculated using equation (1), and the various values are set out in Table 4.

For Fig. 4a, the following expressions can be developed:

$$I_1 = \frac{I(2r_2 + R)(r_1 + R)}{(2r_2 + R)(r_1 + R) + (r_2 + R)[r_1(2+a) + R(1+a)]} \qquad ..(2)$$

$$I_2 = \frac{I(r_2 + R)[r_1(2+a) + R(1+a)]}{(2r_2 + R)(r_1 + R) + (r_2 + R)[r_1(2+a) + R(1+a)]} . \qquad ..(3)$$

The output from this bridge is given by:

$$V_o = I_1 R(1+a) - I_2 R$$

which, when the above expressions for I_1 and I_2 above are substituted, becomes:

$$V_o = \frac{IR\ [R(r_2 - r_1) + r_2 a(r_1 + R)]}{(2r_2 + R)(r_1 + R) + (r_2 + R)[r_1(2+a) + R(1+a)]} . \qquad ..(4)$$

Equation (4) can be used to compare outputs from the circuits of Figs. 4a and 4b by using r_1 and r_2 to calculate V_o, and then using r_1' and r_2' to calculate V_o'.

The values used for r_1, r_2 and r_1', r_2' are shown in Table 4, and the calculated values of V_o/IR are set out in Table 5. It can be seen that the differences for corresponding conditions in the two bridges are negligible.

TABLE 1.

BRIDGE OUTPUT VOLTAGE FORMULAE

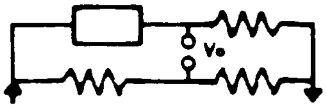
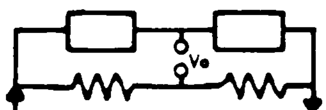
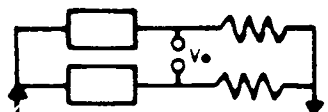
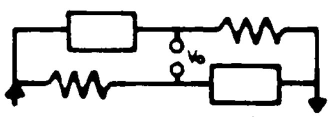
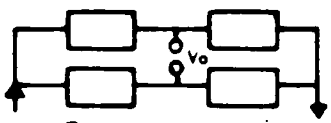
Configuration	Constant current	Constant voltage
 One active arm	$V_o = \frac{IR\delta}{4+\delta}$	$V_o = \frac{V\delta}{4+2\delta}$
 Two arms active	$V_o = \frac{IR\delta}{2}$	$V_o = \frac{V\delta}{2}$
 Two arms active	$V_o = \frac{IR\delta}{2}$	$V_o = \frac{2V\delta}{4-\delta^2}$
 Two arms active	$V_o = \frac{IR\delta}{2}$	$V_o = \frac{V\delta}{2+\delta}$
 Four arms active	$V_o = IR\delta$	$V_o = V\delta$

TABLE 2

RELATIVE SENSITIVITIES FOR DIFFERENT AMOUNTS OF OFFSET COMPENSATION (b)

	C_1			C_2		
$a = \delta$	b +0.001	+0.005	+0.01	b −0.001	−0.005	−0.01
+0.001	0.99999975	0.99999503	0.99998085	0.99949975	0.99749498	0.99498050
+0.003	0.99999963	0.99999441	0.99997962	0.99949963	0.99749436	0.99497925
+0.006	0.99999944	0.99999317	0.99997777	0.99949944	0.99749342	0.99497737
+0.01	0.99999919	0.99999224	0.99997531	0.99949919	0.99749218	0.99497487
−0.001	0.99999988	0.99999565	0.99998208	0.99949988	0.99749561	0.99498175
−0.003	1.00000000	0.99999627	0.99998332	0.99950000	0.99749624	0.99498301
−0.006	1.00000019	0.99999721	0.99998518	0.99950019	0.99749718	0.99498489
−0.01	1.00000044	0.99999986	0.99998766	0.99950044	0.99749843	0.99498741

REFER APPENDIX ONE, EQUATIONS 12 AND 13

TABLE 3

INFLUENCE OF LEAD RESISTANCE ON CALIBRATE SIGNAL

Refer Appendix 2, Equation (6)

R_ℓ OHMS	PERCENT ERROR IN SHUNT CALIBRATE SIGNAL	
	R_{CAL}=10K OHMS	R_{CAL}=200K OHMS
0	0	0
0.2	0.6638	0.6676
0.4	1.3297	1.3373
0.6	1.9979	2.0094
0.8	2.6681	2.6836
1.0	3.3406	3.3601

TABLE 4

REFER APPENDIX 3

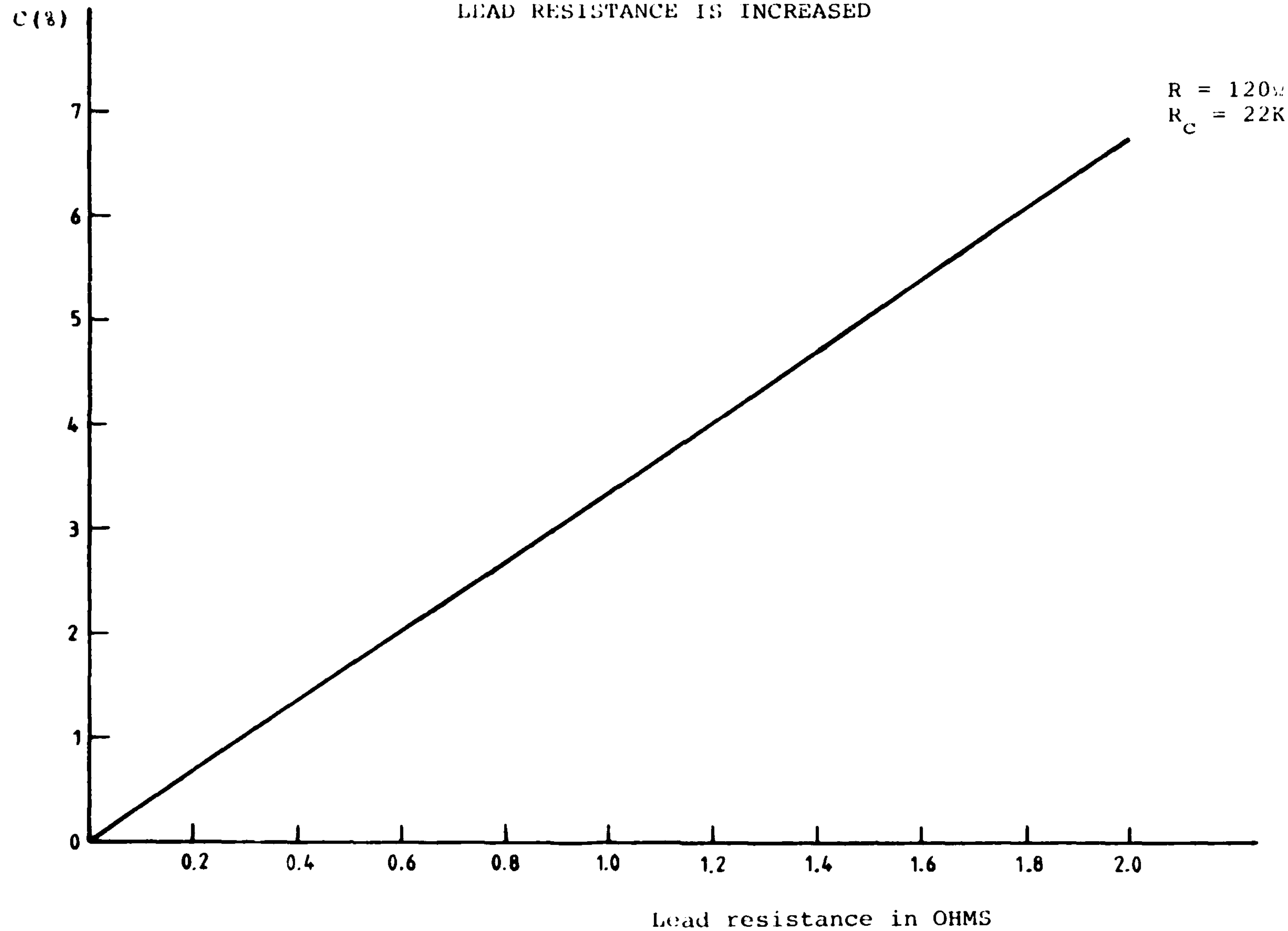
R_{CAL}	R_{SHNT}	'a'	r_1	r_2	r_1'	r_2'
10K	10K	0.012	10002	10002	10003	10003
10K	200K	0.0006	10002	200002	10002.05	200022
200K	10K	0.012	200002	10002	200022	10002.05
200K	200K	0.0006	200002	200002	200003	200003

TABLE 5

REFER APPENDIX 3

r_1	r_2	V_o/IR
10002	10002	2.97313×10^{-3}
10003	10003	2.97314×10^{-3}
10002	200002	2.97265×10^{-3}
10002.05	200002	2.97265×10^{-3}
200002	10002	1.50518×10^{-4}
200022	10002.05	1.50518×10^{-4}
200002	200002	1.49933×10^{-4}
200003	200003	1.49933×10^{-4}

GRAPH 1. PERCENTAGE CHANGE IN CALIBRATION SIGNAL AS LEAD RESISTANCE IS INCREASED



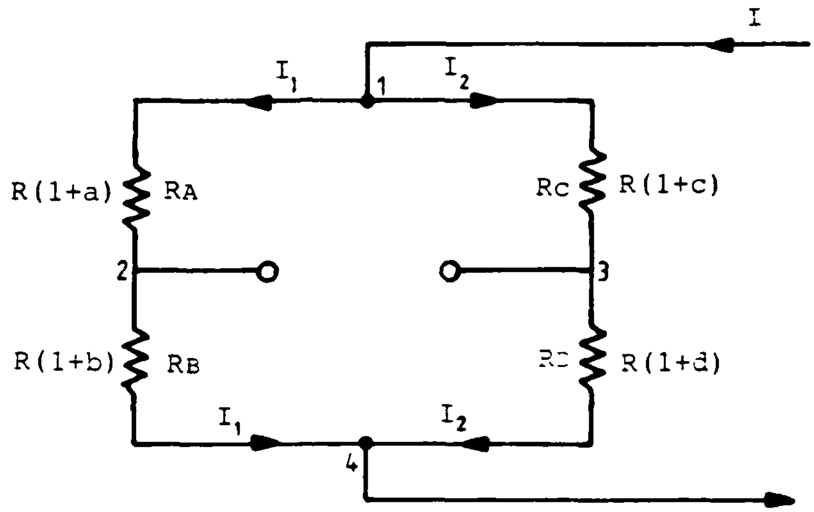


Fig. 1.

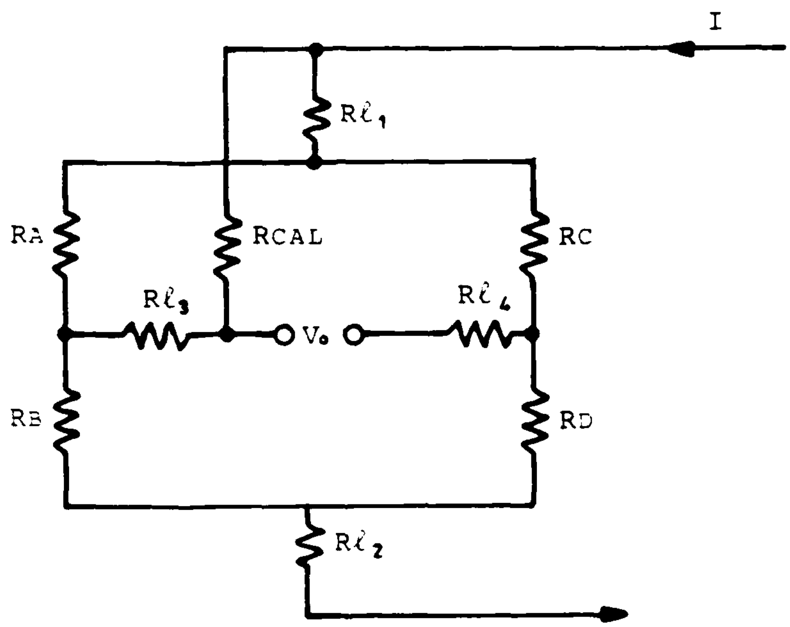


FIG. 2.

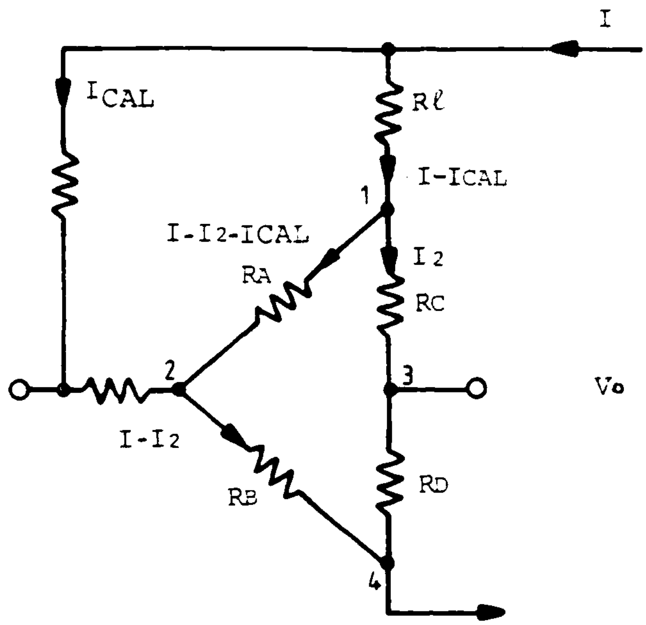


FIG. 3(a)

Delta → star transformation

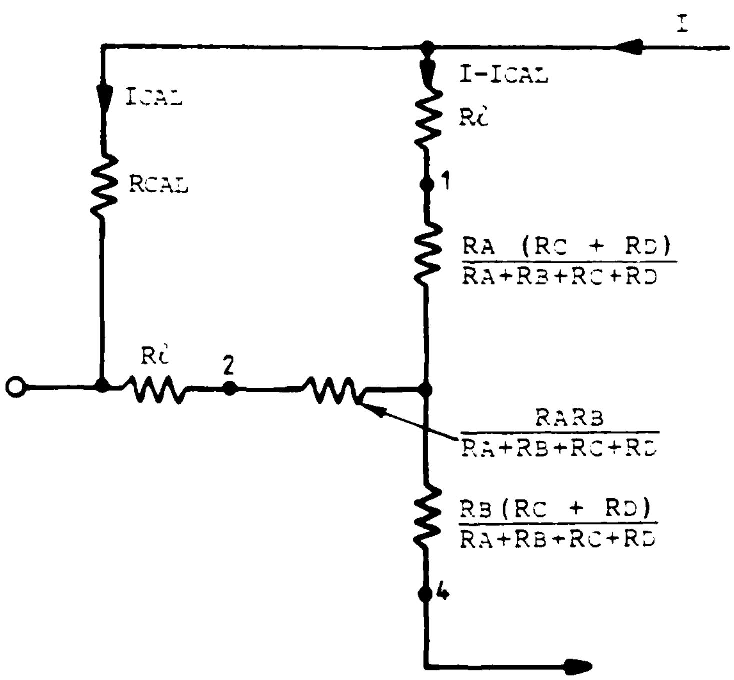


FIG. 3(b)

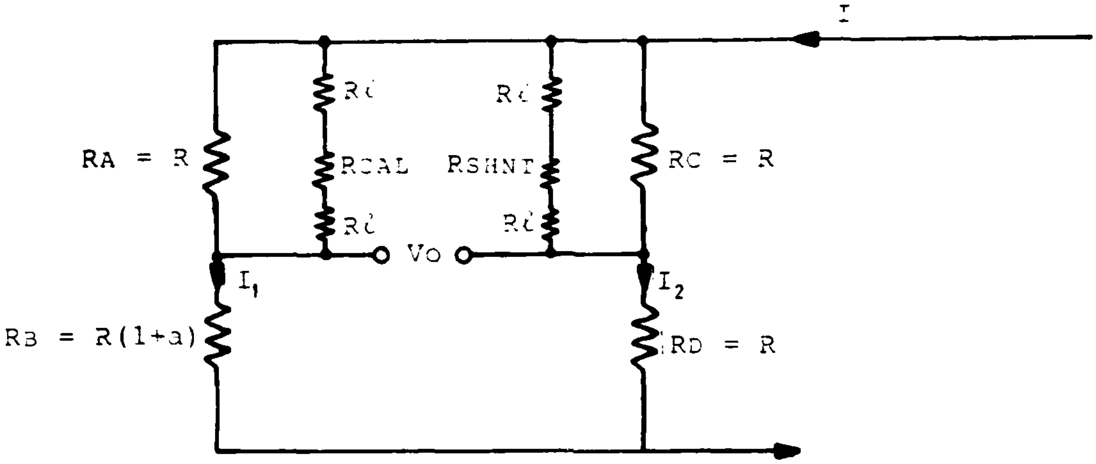


FIG. 4(a)

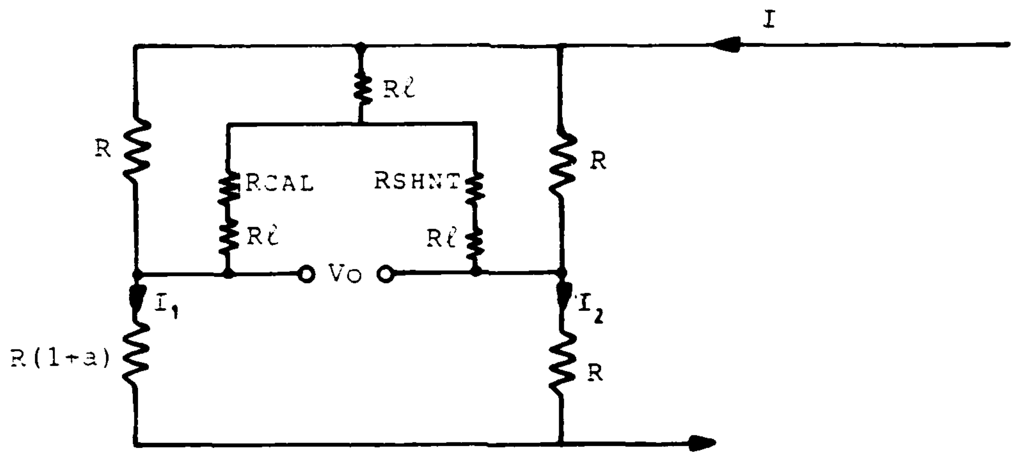


FIG. 4(b)

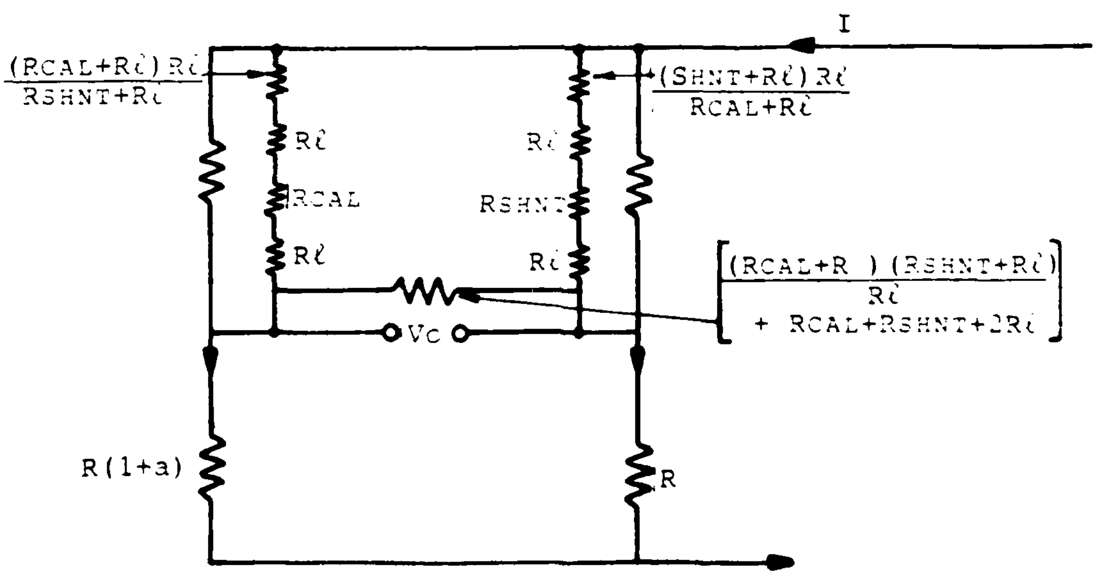


FIG. 4(c)

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16. Abstract
A simplified analysis is used to develop expressions for the output of the commonly used strain gauge bridge configurations with Constant Current excitation. Expressions for initial offset compensation, shunt calibration and the influence of lead resistance are developed. Consideration is given to some means for error correction.

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